Hale School

Mathematics Specialist

Term 1 2018

Test 2 - Functions

Name: MARK I. N.G. KEY

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Instructions:

- **Calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 6% of the year (school) mark**

Question 1 **(6 marks)**

Consider the function $f(x)=\dfrac{3}{(x-1)^2}+6$.

(a) Prove that $f(x)$ is not a one – one function. (2 marks)

$f(2) = \frac{3}{1}+6 = 9$ ✓ evidence

$f(0) = \frac{3}{1}+6 = 9$

$\therefore f(x)$ is many-one and not one-one ✓ statement

(b) State the largest value of a for which $f(x)$ over the domain $\{x : x \le a, x \in \mathbb{R}\}$ is a one-one function. (1 mark)

$a = 1$ ✓ answer

(c) For the domain in part (b), find, $f^{-1}(x)$, the inverse function of $f(x)$. (3 marks)

Inverse is $x = \dfrac{3}{(y-1)^2}+6$ ✓ method - replaces x and y and attempt to rearrange

$x-6 = \dfrac{3}{(y-1)^2}$

$(y-1)^2 = \dfrac{3}{x-6}$

$y-1 = \pm\sqrt{\dfrac{3}{x-6}}$

$y = 1 \pm\sqrt{\dfrac{3}{x-6}}$ ✓ finds $y = \pm$ ..

$\therefore f^{-1}(x) = 1 - \sqrt{\dfrac{3}{x-6}}$ ✓ correct $f^{-1}(x)$

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Question 2 **(7 marks)**

If $g(x)=(x+2)^2$ and $h(x)=\dfrac{1}{3x-1}$, find:

(a) $h\circ g(-3)$ (1 mark)

$h\circ g(-3) = h(1) = \frac{1}{2}$ ✓ answer

(b) the natural domain of $h\circ g(x)$ (3 marks)

$h\circ g(x) = \dfrac{1}{3(x+2)^2-1}$ ✓ $3(x+2)^2-1=0$

$D_{hg} = \{x : x\in R, x \ne -2 \pm \frac{1}{\sqrt{3}}\}$ ✓ $x = -2 \pm 1/\sqrt{3}$

✓ states domain

(c) the natural range of $h\circ g(x)$ (3 marks)

$(x+2)^2 \ge 0$

$\therefore 3(x+2)^2-1 \ge -1$

$\therefore \dfrac{1}{3(x+2)^2-1} \in (-\infty, -1]$ or $(0, \infty)$

$\therefore R_{hg} = \{y : y \le -1, y\in R\} \cup \{y : y>0, y \in R\}$

✓ Finds $y>0$

✓ Finds $y \le -1$

✓ states range correctly

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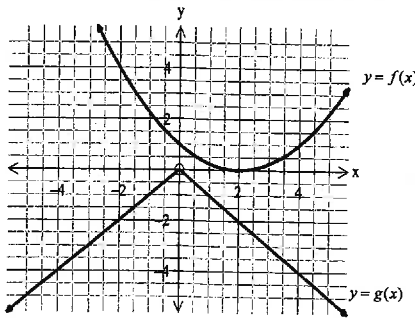
Question 3 **(7 marks)**

The axes below shows the graphs of $y=f(x)$ and $y=g(x)$



(a) Find the values of

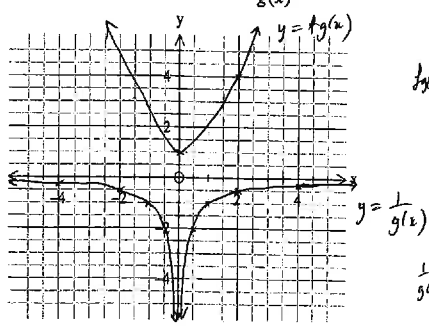
i) $f(g(2)) = f(-2) = 4$ ✓ answer (1 mark)

ii) a so that $g(f(a))=-1$ (2 marks)

$f(a) = -1$ or 1 ✓ $f(a) = \pm 1$

$\therefore a = 0$ or 4 ✓ $a = 0, 4$

(b) On the axes below draw the graphs of $\dfrac{1}{g(x)}$ and $fg(x)$ (4 marks)



$fg(x)$ ✓ $x \le 0$ values ✓ $x \ge 0$ values

$\frac{1}{g(x)}$ ✓ vertical asymptote ✓ horizontal asymptote & joined up

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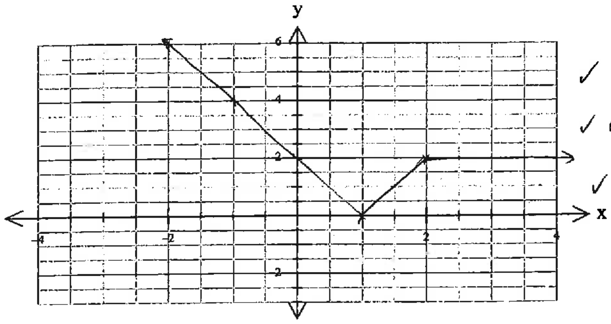
Question 4 **(6 marks)**

(a) On the axis below accurately sketch the graph of $y=\left|x-|x-2|\right|$. (3 marks)

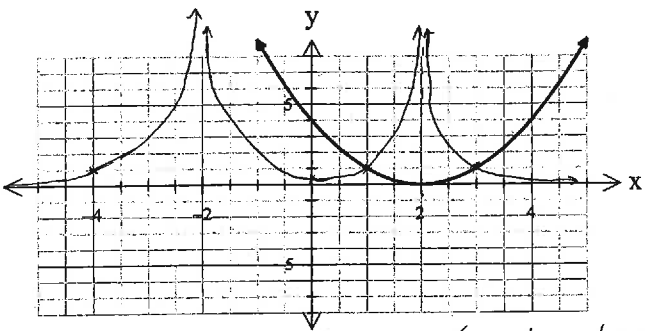


✓ (1, 0)

✓ (0,2) and (2,2)

✓ lines

(b) The graph of $y=f(x)$ has been drawn on the axes below. On the same axes draw an accurate sketch of the graph of $y=\dfrac{1}{f(|x|)}$. (3 marks)



✓ symmetry about y-axis

✓ vertical asymptotes at $x = \pm 2$

✓ accurate graph (including where $y = 1$)

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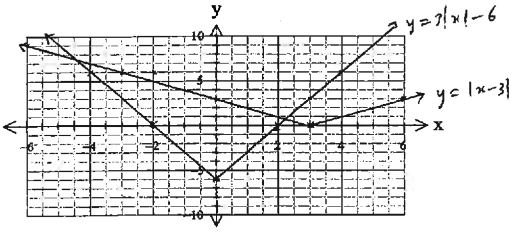
Question 5 **(7 marks)**

Consider the functions $f(x)=3|x|-6$, $g(x)=|x-3|$ and $h(x)=m|x|+b$

(a) On the axes below, draw the graphs of $y=f(x)$ and $y=g(x)$ (2 marks)



✓ $y = |x-$

✓ $y = 3|x|-$

(b) Determine the exact values of x for which $f(x)=g(x)$. (2 marks)

Meet when $3 - x = 3x - 6$ or $3 - x = -3x + 6$

$9 = 4x$ $\quad$ $2x = -9$

$x = 9/4$ $\quad$ $x = -9/2$

$f(x) = g(x)$ when $x = \frac{-9}{2}$ or $x = \frac{9}{4}$

✓ $x = -9/2$

✓ $x = 9/4$

(c) State the values of m, b and k for which the solution set for the equation $h(x)=g(x)$ is $\{x : x\in\mathbb{R}, 0\le x\le k\}$ (3 marks)

Need the same gradient $\Rightarrow m = \pm 1$

Overlaps from 0 to k

$\therefore h(x) = -|x| + 3$

$\therefore m = -1, \quad b = 3, \quad k = 3$

✓ m =

✓ b =

✓ k =

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Question 6 **(4 marks)**

The graph of $y=\dfrac{ax^2+bx+4}{x-c}$ has an oblique asymptote of $y=2x-1$ and a vertical asymptote at $x=3$. Determine the values of a , b and c.

$$\frac{ax^2+bx+4}{x-c} = 2x - 1 + \frac{d}{x-c}$$

✓ c = 3

$$ax^2+bx+4 = 2x^2 - (1+2c)x + c + d$$

✓ rearrange

$\therefore$ $a = 2$, $c = 3$, $b = -7$

✓ a = 2

✓ b = −7

$$y = \frac{2x^2 - 7x + 4}{x - 3}$$

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